Energy Conservation Errors of Objective Stress Rates in ABAQUS, ANSYS, LS-DYNA and Other FE Codes: Their Magnitude and How to Correct Them

kterou prosloví

Zdeněk P. Bažant
Northwestern University, Evanston, Illinois, USA

Abstract

The objective stress rates used in most commercial finite element (FE) codes for the implicit incremental (updated Lagrangian or Riks) analysis is the Jaumann rate of Cauchy (true) stress. However, if the material is compressible, this rate violates the law of conservation of energy because it is not work-conjugate to any finite strain tensor. For bifurcation load analysis, most programs use the Jaumann rate of Kirchhoff stress. Work-conjugate though it is to the Hencky (or logarithmic) strain tensor, in the case of buckling of highly orthotropic soft-in-shear structures it causes another kind of energy conservation error — incorrect work of initial stresses. For such structures, only the Truesdell stress rate which is work-conjugate to the Green-Lagrangian finite strain tensor is correct (it is used in commercial code ATENA and open-source code OOFEM, both developed in Prague). The Green-Naghdi rate is not work-conjugate to any finite strain tensor and can be associated with a tangential material stiffness matrix that lacks major symmetry although, in commercial explicit programs, this rate in used in a symmetrized form.

These problems with work-conjugacy were, in principle, theoretically demonstrated in 1971. Nonetheless, they have generally been ignored, partly because of some extraneous considerations, partly because in the vast majority of applications, mainly to metals, the errors are negligible. However, with the growing interest in highly compressible materials, soft-in-shear orthotropic materials and quasibrittle materials with softening damage, large discrepancies have been noticed. Often, though, they have been regarded as a matter of choice depending on the type of material or application, rather than as problems with the first law of thermodynamics.

Numerical examples document that the errors can be serious. One is the indentation of a naval-type sandwich plate with a polymeric foam core, in which the error amounts to 29% of the indentation force and 15% in the work of load. Errors from 40% to 100% are demonstrated for the buckling of homogenized sandwich panels planned for a novel type of ribbed ship hulls of super-light and long ships. Errors of similar kind must be expected for all highly compressible materials, such as metallic and ceramic foams, fiber reinforced foam cores, honeycomb, loess, silt, organic soils, pumice, tuff, corral, light wood, osteoporotic bone and various biologic tissues, and for compression damage models in which the material is rendered incrementally highly orthotropic due to dense axial splitting cracks.

A remedy can be achieved if the previously derived equations relating the tangential moduli tensors associated with the Jaumann rates of Cauchy or Kirchhoff stresses and with the Truesdell rate are used in the user's material subroutine of a black-box implicit commercial

program. However, these corrections must be delayed, which accumulates additional numerical integration error. A better remedy would be a revision of the black-box commercial programs.

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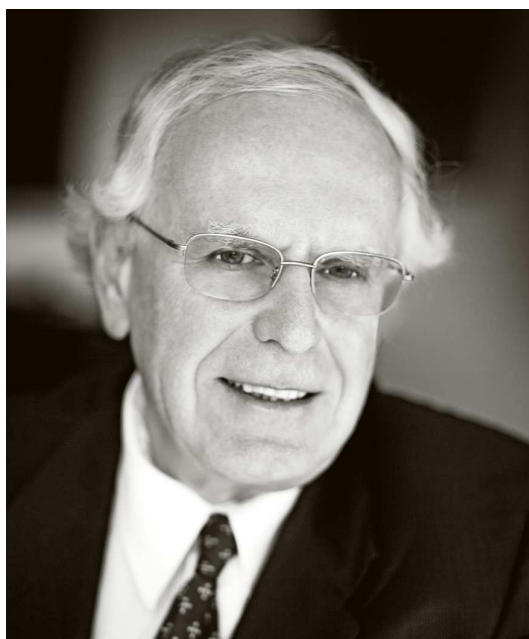
Born and educated in Prague (Ph.D. 1963), Bažant joined Northwestern in 1969, where he has been W.P. Murphy Professor since 1990 and simultaneously McCormick Institute Professor since 2002, and Director of Center for Geomaterials (1981-87). He was inducted to Nat. Academy of Sciences, Nat. Academy of Engrg. and Am. Acad. of Arts & Aci., as well as Italian Nat. Acad. (dei Lincei), Austrian Acad. of Sciences, Czech Acad. of Engrg., Spanish Royal Acad. of Eng., Eur. Acad. of Sci. & Arts, and Istituto Lombardo. An Illinois Registered Structural Engineer, he is Hon. Mem. of ASME, ASCE, ACI & ČSM. He received 7 honorary doctorates (Prague, Karlsruhe, Colorado, Milan, Lyon, Vienna, Ohio State); ASME Timoshenko, Warner and Nadai Medals; ASCE von Karman, Newmark, Biot and Croes Medals and Lifetime Achievement Award; SES Prager Medal; Am. Ceramic Soc. Roy Award; ICOSSAR Lecture Award; RILEM L'Hermite Medal; Šolín, ČSM and Bažant, Sr. Medals in Prague; Exner Medal, Vienna; etc. He published 6 books: Scaling of Structural Strength, Inelastic Analysis, Fracture and Size Effect, Stability of Structures, Concrete at High Temperatures, and Concrete Creep and works with M. Jirásek on his 7th. With H-index 71 and ~23,000 citations (on Google), he is one of the original top 100 ISI Highly Cited Scientists in engineering. (http://www.isihighlycited.com/)