

27th Colloquium Lecture, School of Mathematics Faculty of Mathematics and Physics

Prof. Jesper Møller

Aalborg University

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Sokolovská 83, Praha 8 – Karlín

Singular distribution functions constructed from stationary time series and point processes

Abstract

Joint with Horia Cornean, Ira W. Herbst, Benjamin Støttrup, and Kasper S. Sørensen

Consider the cumulative distribution function (CDF) F of a random variable $X := \sum_{n \geq 1} X_n q^{-n}$, where each random variable $X_n \in \{0, \dots, q-1\}$ for a given integer $q \geq 2$; if $q = 2$ we identify the times series $\{X_n\}_{n \geq 1}$ by the point process $Y := \{n \mid X_n = 1\}$. We show that stationarity of $\{X_n\}_{n \geq 1}$ (or Y when $q = 2$) is equivalent with a certain functional equation obeyed by F , which enables us to give a complete characterization of the structure of F . In particular, we prove that the absolutely continuous component of F can only be the uniform distribution on the unit interval while its discrete component can only be a countable convex combination of certain explicitly computable CDFs for distributions with finite support. Moreover, we show that for a large class of stationary time-series, their corresponding F is singular (that is, $F' = 0$ almost everywhere) and continuous; and often also strictly increasing. Finally, we study special cases of models, including stationary Markov chains of any finite order, stationary renewal point processes, and mixtures of such models, where expressions and plots of F will be exemplified.

Further information

<http://msekc.karlin.mff.cuni.cz/colloquia>